Expected utility and risk premium

The vNM utility function of an agent is given by $v(x) = \sqrt{x}$. The agent faces two lotteries, $L_1 = (64, 16)$ with probabilities $(1/4, 1/3)$ and $L_2 = (25)$ with probability 1

1. Which lottery is chosen by the agent?
2. Determine the risk premium of L_1.
3. Explain the risk premium.

Solutions

1. The expected utilities are $EU(L_1) = \frac{1}{4}\sqrt{64} + \frac{3}{4}\sqrt{16} = 2 + 3 = 5$ and $Eu(L_2) = 1\sqrt{25} = 5$. Since $EU(L_1) = Eu(L_2)$, the agent is indifferent between the two lotteries. So, either L_1 or L_2 is chosen by the agent.

2. The expected payoff of L_1 is given by $E(L_1) = \frac{1}{4} \times 64 + \frac{3}{4} \times 16 = 16 + 12 = 28$. The certainty equivalent of L_1 is given by $CE(L_1) = 25$ because $Eu(L_1) = Eu(L_2)$. So, the risk premium is given by $RP(L_1) = E(L_1) - CE(L_1) = 28 - 25 = 3$.

3. The risk premium is a concept in economics and finance that refers to the additional return or premium that an investor requires to hold a risky asset rather than a risk-free asset. It is essentially the amount an individual would be willing to pay to avoid taking on risk.

   In the context of this problem, the risk premium of lottery L_1 is calculated as the difference between its expected payoff and its certainty equivalent. The expected payoff of L_1 is the average value that the agent would receive if they played the lottery many times. The certainty equivalent, on the other hand, is the sure amount of money that the agent considers equally desirable as the risky lottery.

   The risk premium of L_1 was calculated to be 3. This means that the agent would be willing to pay up to 3 units of money to avoid the risk inherent in lottery L_1. In other words, the agent would be indifferent between receiving a sure amount of 25 units of money and playing the lottery L_1 with an expected payoff of 28 units of money. This reflects the agent's attitudes towards risk, and in this case, it shows that the agent is risk-averse.